

The College Board
Advanced Placement Examination
CALCULUS AB
SECTION II
Time—1 hour and 30 minutes

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

1. Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$.
 - (a) Find the zeros of f .
 - (b) Write an equation of the line tangent to the graph of f at $x = -1$.
 - (c) The point (a, b) is on the graph of f and the line tangent to the graph at (a, b) passes through the point $(0, -8)$ which is not on the graph of f . Find the values of a and b .

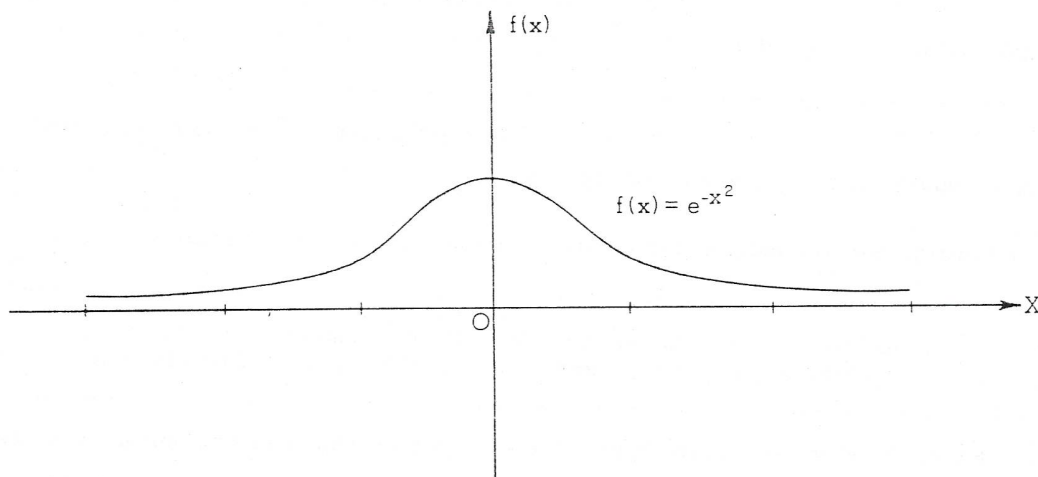
2. Let $f(x) = (1 - x)^2$ for all real numbers x , and let $g(x) = \ln x$ for all $x > 0$. Let $h(x) = (1 - \ln x)^2$.
 - (a) Determine whether $h(x)$ is the composition $f(g(x))$ or the composition $g(f(x))$.
 - (b) Find $h'(x)$.
 - (c) Find $h''(x)$.
 - (d) On the axes provided, sketch the graph of h .

3. Given the function f defined by $f(x) = \frac{2x - 2}{x^2 + x - 2}$.
 - (a) For what values of x is $f(x)$ discontinuous?
 - (b) At each point of discontinuity found in part (a), determine whether $f(x)$ has a limit and, if so, give the value of the limit.
 - (c) Write an equation for each vertical and horizontal asymptote to the graph of f . Justify your answer.
 - (d) A rational function $g(x) = \frac{a}{b + x}$ is such that $g(x) = f(x)$ wherever f is defined. Find the values of a and b .

4. A particle moves on the X -axis so that its velocity at any time t is given by $v(t) = \sin 2t$. At $t = 0$, the particle is at the origin.
 - (a) For $0 \leq t \leq \pi$, find all values of t for which the particle is moving to the left.
 - (b) Write an expression for the position of the particle at any time t .
 - (c) For $0 \leq t \leq \frac{\pi}{2}$, find the average value of the position function determined in part (b).

5. Given the curve $x^2 - xy + y^2 = 9$.
- Write a general expression for the slope of the curve.
 - Find the coordinates of the points on the curve where the tangents are vertical.
 - At the point $(0, 3)$ find the rate of change in the slope of the curve with respect to x .

6. Given the function f defined by $f(x) = e^{-x^2}$.



- Find the maximum area of a rectangle that has two vertices on the X -axis and two on the graph of f . Justify your answer.
 - Let R be the region in the first quadrant bounded by the X - and Y -axes, the graph of f , and the line $x = k$. Find the volume of the solid generated by revolving R about the Y -axis.
 - Evaluate the limit of the volume determined in part (b) as k increases without bound.
-
7. Let g and h be any two twice-differentiable functions that are defined for all real numbers and that satisfy the following properties for all x :
- $(g(x))^2 + (h(x))^2 = 1$
 - $g'(x) = (h(x))^2$
 - $h(x) > 0$
 - $g(0) = 0$
- Justify that $h'(x) = -g(x)h(x)$ for all x .
 - Justify that h has a relative maximum at $x = 0$.
 - Justify that the graph of g has a point of inflection at $x = 0$.

END OF EXAMINATION